Noise

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Noise Description

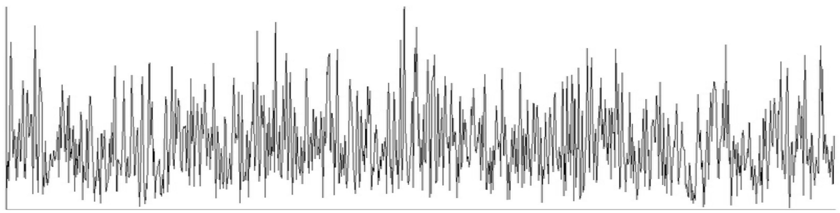
What is Noise ?

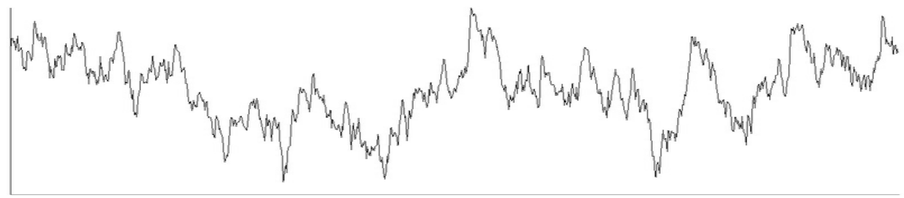
- Noise is a random fluctuation of a voltage / current
- The average noise is zero: $\langle noise \rangle = 0$
  - A non-zero average is no noise, just a 'bias' or 'offset'
- The noise 'strength' can be defined as the variance:

  $$\text{voltagenoise}^2 = \langle v^2 \rangle \quad \text{or} \quad \text{currentnoise}^2 = \langle i^2 \rangle$$

  where $\langle .. \rangle$ is over time
- The 'RMS Noise' is the square root of the variance

- The same RMS can be obtained by very different noise signals, as seen on an oscilloscope (time domain):

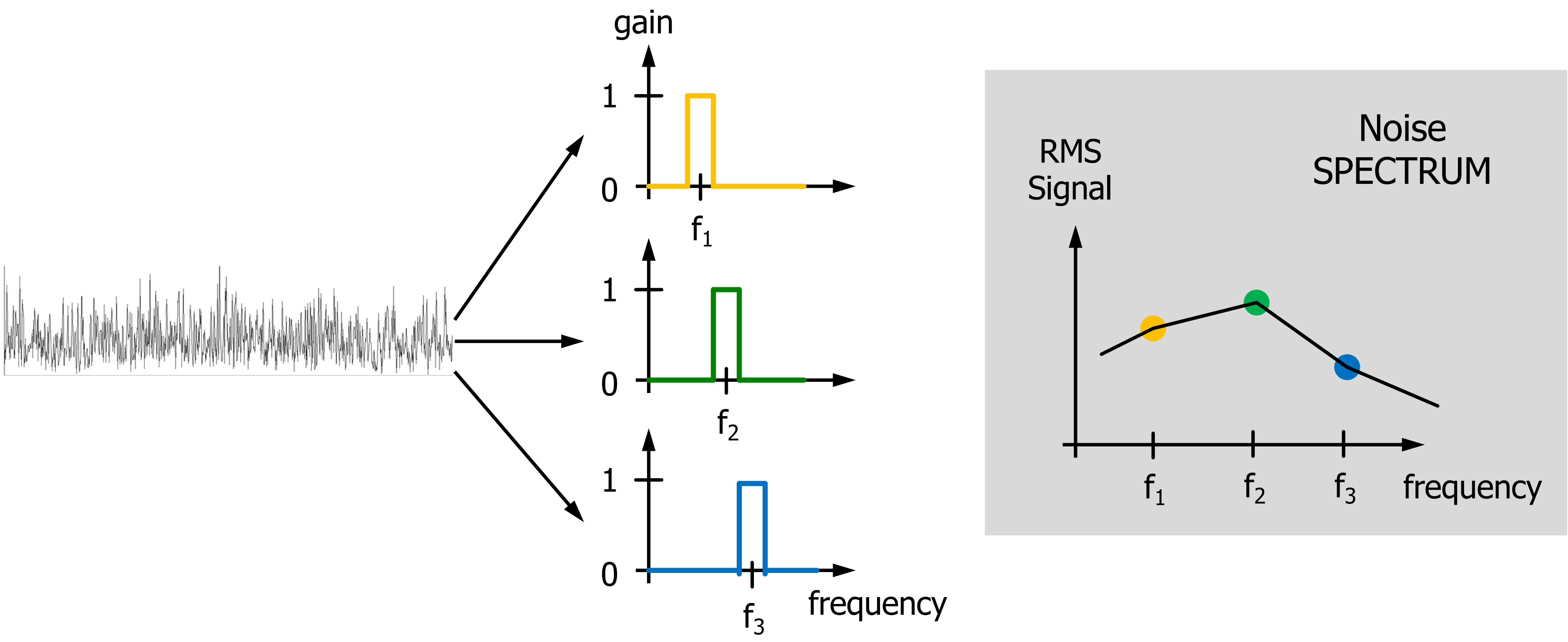




  - Obviously, the left signal contains 'higher frequencies'…

Frequency Composition

- Frequency components can be ‘extracted’ by passing the signal though a (narrow band) (frequency) filter
- This delivers a *noise spectrum*



Spectral Density

- The noisy signal can have different strength for various frequencies.
- We therefore describe noise by its *spectral density*, the (squared) noise voltage (density) as a function of frequency.
  - It has the unit V^2/Hz
  - Sometimes, we use the square root with the unusual unit $V/\sqrt{Hz}$

ANALOG DEVICES

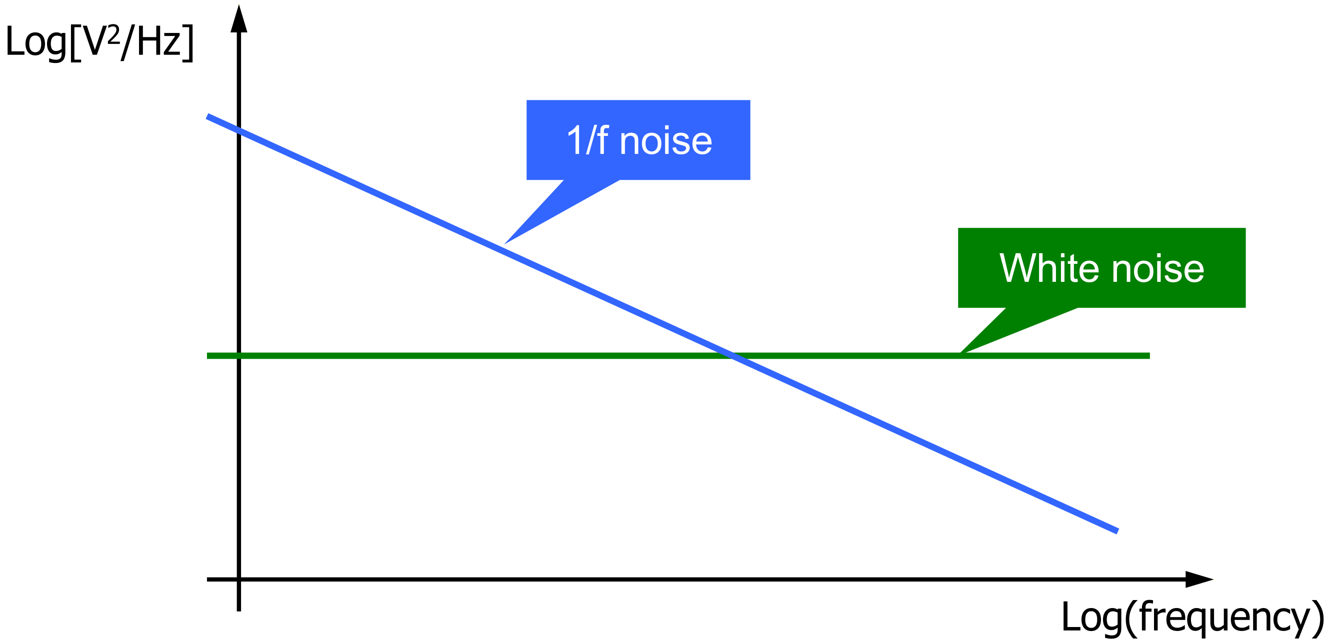
30 V, 8 MHz, Low Bias Current, Single-Supply, RRO, Precision Op Amps

Data Sheet **ADA4622-1/ADA4622-2**

NOISE PERFORMANCE				
Voltage Noise	e_N p-p	0.1 Hz to 10 Hz	0.75	µV p-p
Voltage Noise Density	e_N	f = 10 Hz	30	nV/√Hz
		f = 100 Hz	15	nV/√Hz
		f = 1 kHz	12.5	nV/√Hz
		f = 10 kHz	12	nV/√Hz
Current Noise Density	i_N	f = 1 kHz	0.8	fA/√Hz

Noise Types / Spectra

- Most common types are
  - *White noise* has *constant* spectral density
  - *1/f* noise (*pink* noise) spectrum is ~ 1/f (or $S(f) \propto 1/f^{\alpha}$)
- Be careful: one can use frequency ν, or angular freq. ω!



- The (total) rms noise is the integral of the noise spectral density over all frequencies (ν = 0…∞)

NOISE IN COMPONENTS

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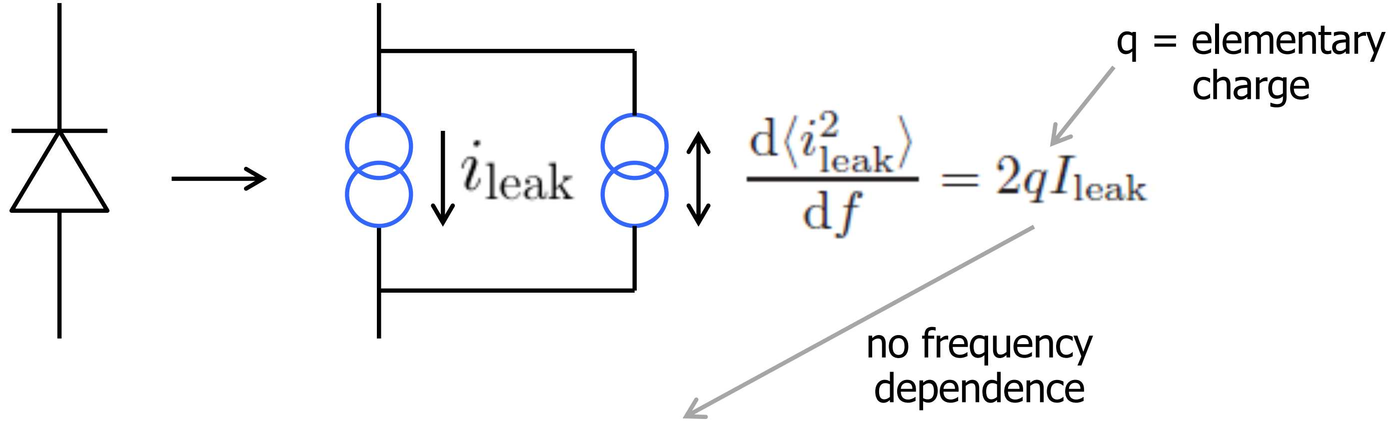
Noise in Diodes

- The reverse current ('leakage') of a diode is generated by charge carriers which statistically overcome a barrier.
- The statistical fluctuations lead to noise.
  The fluctuations depend on the value of the leakage current.
- This is called shot noise.

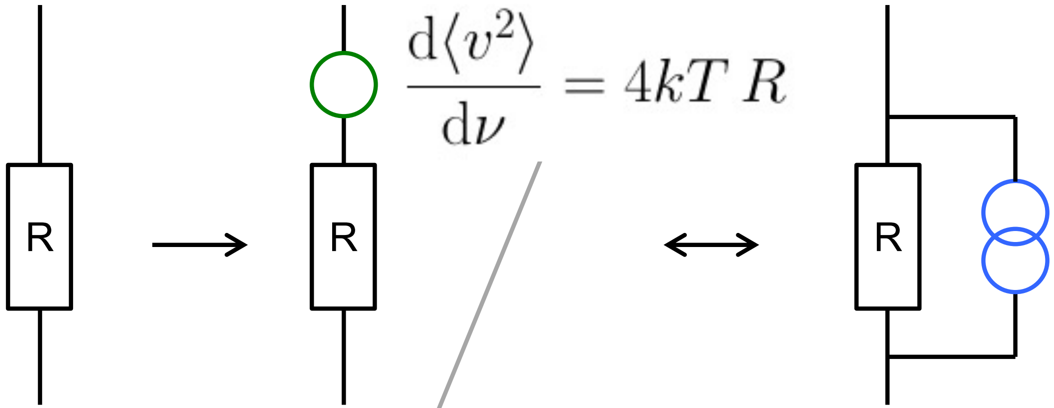


- Spectrum is flat (white noise)

- Check the units: $[2\ q\ I_{Leak}] = C \times A = As \times A = A^2/Hz$

Noise in Resistors

- Resistors exhibit noise from thermal motion of charge carriers. This noise is *independent* on current flow!
- This *thermal noise* is *white* noise
- If can be modelled by a serial voltage source
  OR
  by the Thévenin equivalent parallel current source



$$\frac{d\langle v^2\rangle}{d\nu} = 4kT\,R$$

$$\frac{d\langle i^2\rangle}{d\nu} = \frac{4kT}{R}$$

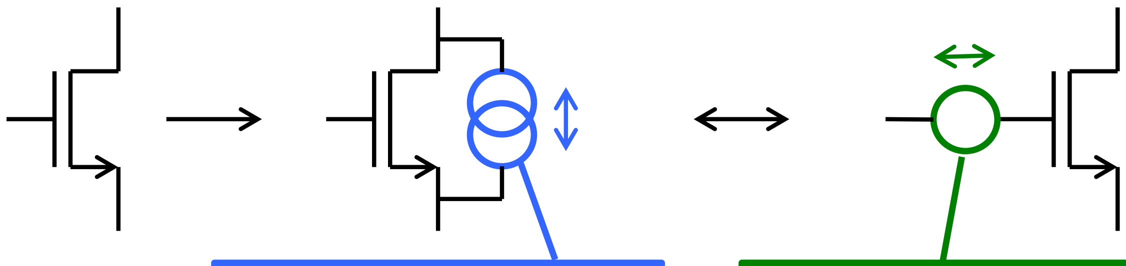
• Check Units: [4kT R] = V A s × V/A = V^2/Hz
[4kT/R] = V A s / V/A = A^2/Hz

$$4kT \approx 1.6 \times 10^{-20}\, V\,As = 16 \times zV\,A/Hz$$

Observe that (only!) these models give correct noise for serial / parallel connections of Rs

White Noise in Transistors

- The MOS channel can be seen as a series of (position dependent) resistor (at least in linear operation). Their noise contributions can be integrated up.
- The white current noise in the channel is $\frac{\mathrm{d}\langle i^2\rangle}{\mathrm{d}\nu} = 4kT\,\gamma\,g_m$
  - the factor γ from integration varies depending on operation regime: γ = 2/3 in strong inversion, less in w.i.
- This current noise *at the drain* can also be written as a voltage noise *at the gate* (by dividing by g_m^2)



$$\frac{\mathrm{d}\langle i^2\rangle}{\mathrm{d}\nu} = 4kT\,\gamma\,g_m$$

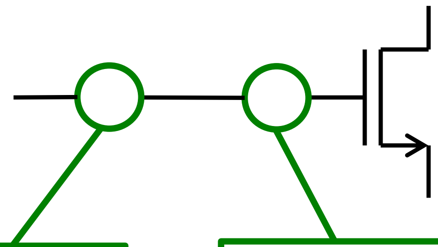
$$\frac{\mathrm{d}\langle v^2\rangle}{\mathrm{d}\nu} = \frac{4kT\,\gamma}{g_m} \approx \frac{8}{3}\frac{kT}{g_m}$$

- Note: noise (i.e. γ) for very short devices can be increased

1/f Noise in Transistors

- Charge carriers in the channel can be captured ('trapped') at impurities and released later.
  - This happens mostly at the oxide interface
- This leads to an additional noise with *1/f spectrum*
- The importance of this contribution depends on
  - Fabrication process → Technology noise parameter $\mathbf{K_f}$
  - MOS type (JFETs are *very much* (>10 x) better (no Interface))
  - MOS polarity (PMOS are *significantly* (10 x) better than NMOS)
- The effect averages out for larger devices (→WL in formula)

Independent of temperature (as long as traps do not freeze out)



$$\frac{\mathrm{d}\langle v_{1/\mathrm{f}}^2\rangle}{\mathrm{d}f} = \frac{K_\mathrm{f}}{C_\mathrm{ox} W L}\frac{1}{f}$$

1/f noise

$$\frac{\mathrm{d}\langle v^2\rangle}{\mathrm{d}\nu} = \frac{8}{3}\frac{kT}{g_m}$$

Thermal noise (white)

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NOISE CALCULATIONS

Recipes

- Noise contributions are independent → Each source is treated separately and noise contributions are added up (in quadrature) at the end
- For each source
  - calculate the transfer function H(s) to the 'output' node
  - multiply the noise spectrum of the source with $|H|^2(s)$ (because we treat squared voltages)
  - integrate the 'output' spectrum over all frequencies.
- Add the resulting variances of all sources
- Square root of the sum leads the final noise (at the output)

- The noise rms (in V or A) value must be compared to the signal (which also depends on H(s)) to get a Signal-to-Noise ratio (SNR)

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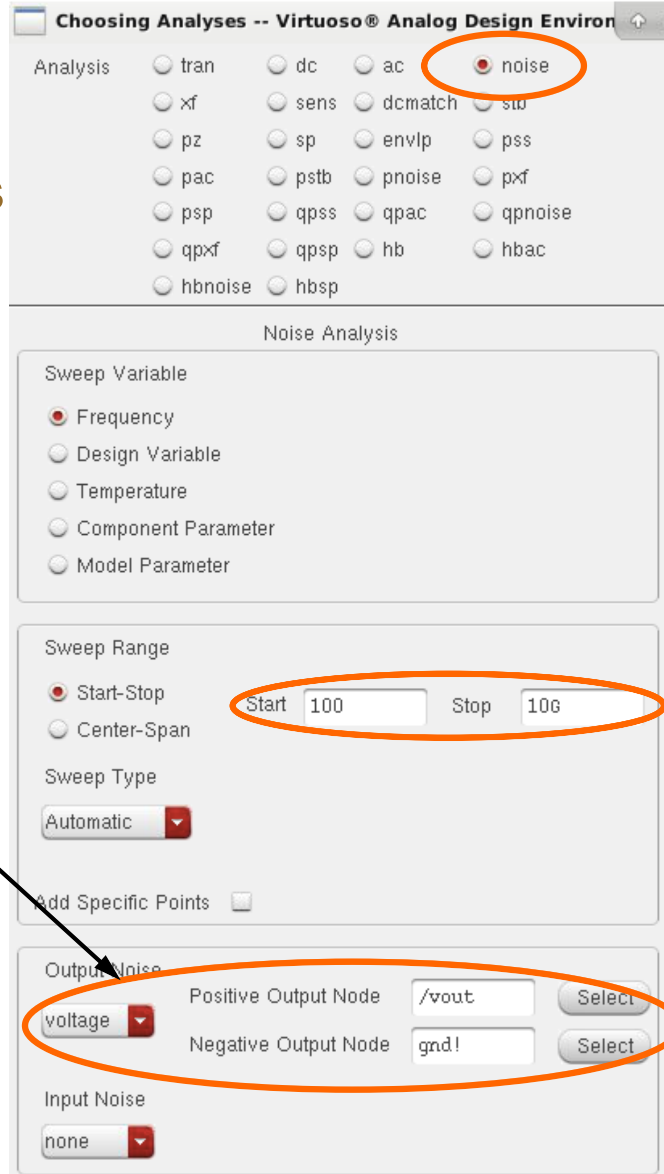
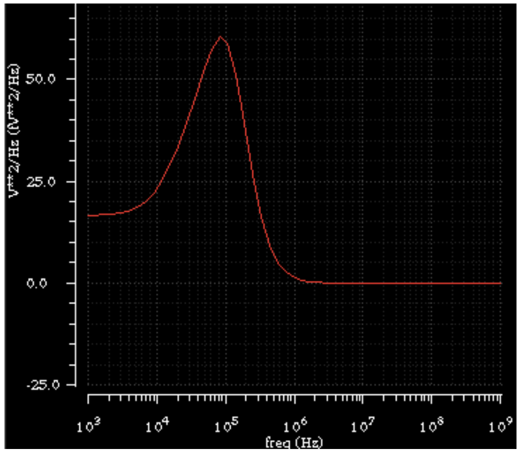
NOISE SIMULATION

Noise Simulation

- Simulation type 'noise' creates AC noise for components
  - Noise can be turned off in parameters
- Give a (generous) frequency range
- Noise is 'collected' for ONE node
  This can be
  - a voltage (give terminals)
  - a current (give a voltage source)
- You can select a node for plotting.
  You will see the V^2/Hz spectrum



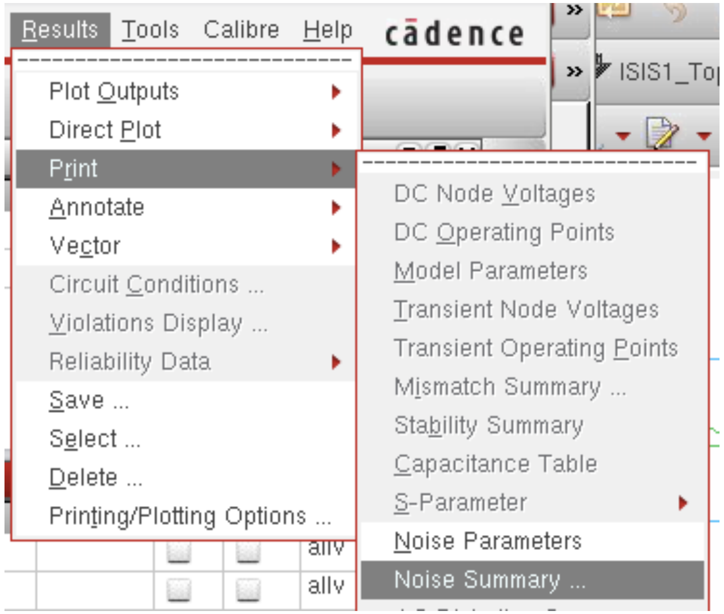


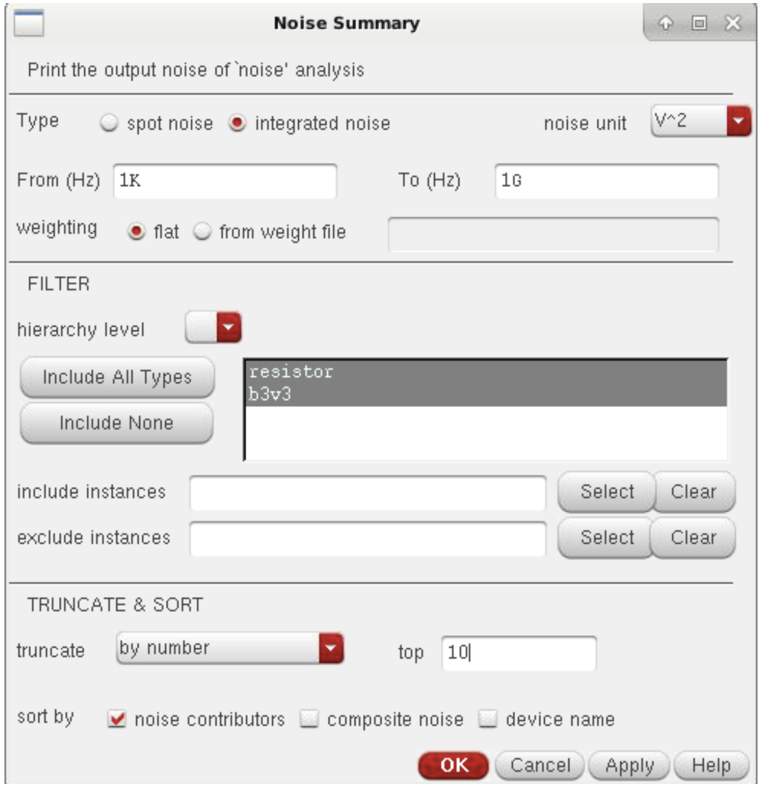
Total Noise

- To integrate a noise spectrum over frequency use the `totalNoise( "noise" nil nil nil)` command in ADE
  - Arguments are **`start_freq stop_freq exlusions`**.
    They can be set to **`nil`** for full range / all components
  - **No** need to specify a net / node
    (the 'output noise' node selected in the simulation window is taken)
- This will give the *squared* noise (in V^2 or A^2)
- To get RMS noise (in V or A), calculate the sqrt(…)!

Noise Summary

- In more complex circuits, you can see which components contribute to the output noise:
  - Results → Print → Noise Summary
  - Usually select 'integrated noise'
  - Set the correct frequency range!





Noise Summary Result

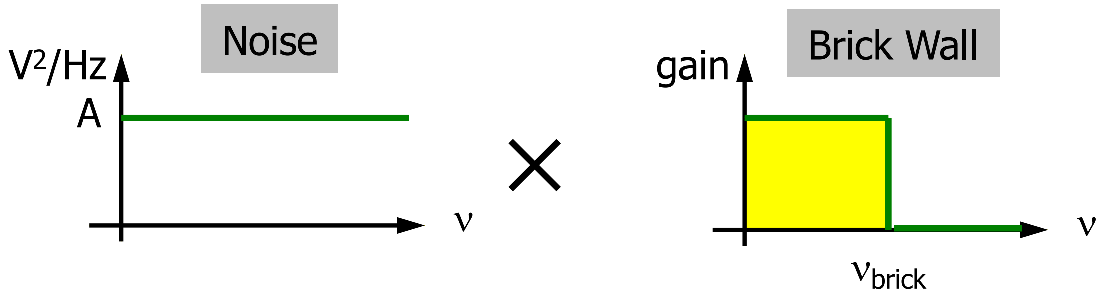
- Lists most important contributions
- Also gives the type of noise:
  - Resistor noise ('rn')
  - Channel noise ('id')
  - 1/f noise ('fn)
  - Source resistive noise ('rs') (depends on actual layout!)
  - Drain resistive noise ('rd') (depends on actual layout!)
  - Gate resistive noise (?) (depends on actual layout!)

Device	Param	Noise Contribution	% Of Total
/M1	fn	1.0337e-08	58.40
/R1	rn	4.17986e-09	23.62
/R0	rn	2.098e-09	11.85
/C2	rn	9.66773e-10	5.46
/M1	id	7.17454e-11	0.41
/MN	id	3.52872e-11	0.20
/MN	fn	1.07926e-11	0.06
/M1	rs	4.07753e-14	0.00
/MN	rs	3.16337e-15	0.00
/M1	rd	2.69697e-17	0.00

NOISE BANDWIDTH LIMITATION

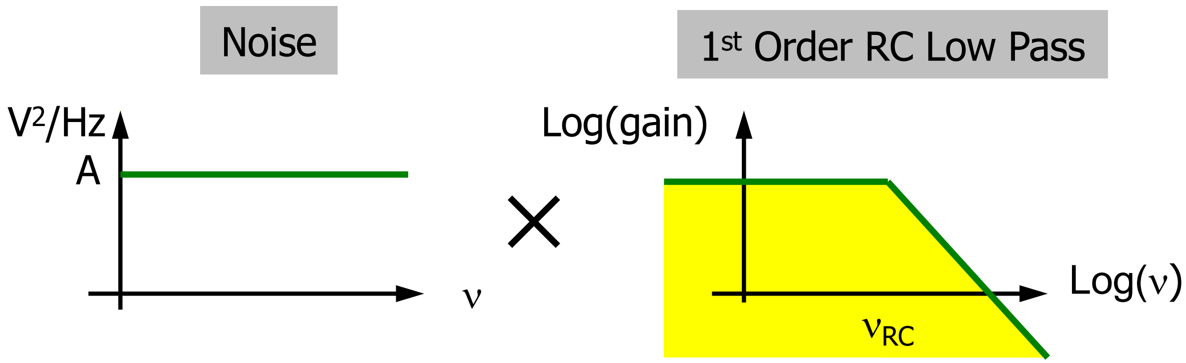
Bandwidth Limitation

- **Brick-Wall Low Pass:**
  - Assume a Low Pass which passes all frequencies up to ν_{brick} and then stops perfectly



  - When we filter white noise, the overall noise is just $\mathbf{rms^2 = A\ \nu_{brick}}$

- **RC Low Pass:**
  - How is the integral now ?



The Integral

- Transfer Function of RC Low Pass: $H(\omega) = \frac{1}{1 + i\,\omega\tau} \quad (\tau = RC)$

- Gain: $v^2(\omega) = |H(\omega)|^2 = H(\omega)H^*(\omega)$

$$= \frac{1}{(1 + i\,\omega\tau)(1 - i\,\omega\tau)} = \frac{1}{1 + (\omega\tau)^2}$$

- Integral: $rms^2 = \int\limits_0^\infty \frac{A}{1 + (2\pi\nu\tau)^2} \mathrm{d}\nu$

$$= \frac{A}{2\pi\tau} \int\limits_0^\infty \frac{\mathrm{d}x}{1 + x^2} = \frac{A}{2\pi\tau}\frac{\pi}{2} = \frac{A}{4\tau} = \frac{A\omega_0}{4} = \frac{A\pi\nu_0}{2}$$

(an integral of $H(\omega) \sim 1/\omega$ would not converge, but $H^2(\omega)$ does)

- To obtain the same noise with a 'brick wall' filter, we need
  - $A\,\nu_{brick} = A\,\pi\,\nu_{RC} / 2 \quad \rightarrow \quad \nu_{brick} = \nu_{RC} \times \pi / 2$

Noise of RC Low Pass

- In the previous calculation, we have assumed R in the Low-Pass as noiseless. In reality, it is noisy
- If we have no *signal* at the input, the remaining input noise is just the *voltage noise* of the resistor, i.e. A = 4kT R
- The resulting output voltage noise is

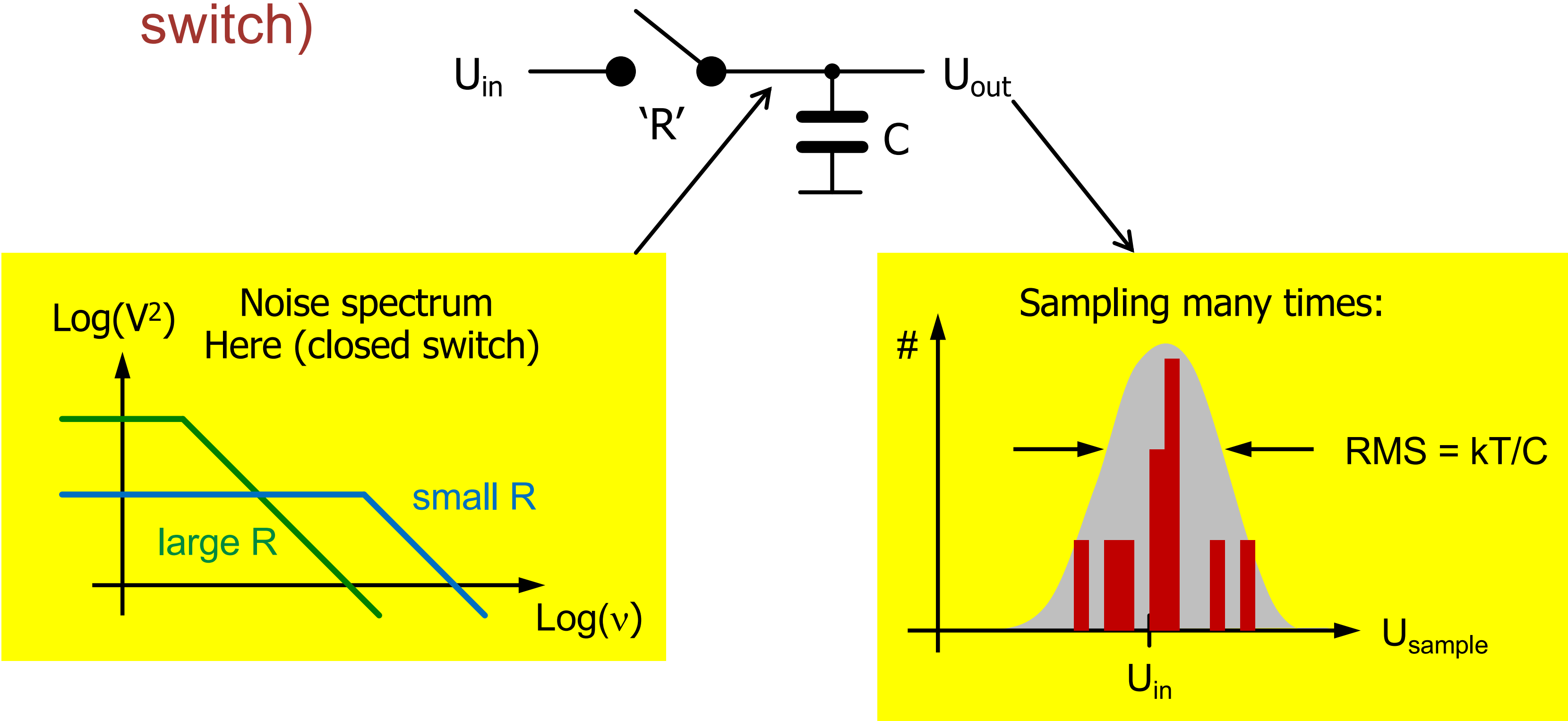
$$rms^2 = \frac{1}{4} A\,\omega = \frac{1}{4}\,4kT\,R\,\frac{1}{RC} = \boxed{\frac{kT}{C}}$$

- 'Charge noise' is <q>^{2}=kTC (from $Q^2=U^2C^2$)
- This 'kT/C-Noise' does *not* depend on R, but on C
  - This is because the change in bandwidth (with R) just compensates the change in noise
  - This kTC noise is present whenever signals are sampled to Cs!

Small capacitors 'have' large (voltage) noise

Sampling Noise

- An analogue voltage U_0 can be stored on a capacitor C. The resistivity R of the switch leads to an RMS noise, independent of the value of R.
- This fluctuation is seen at U_{out} when sampling (opening the switch)



- Precise sampling (voltage storage) needs large storage cap

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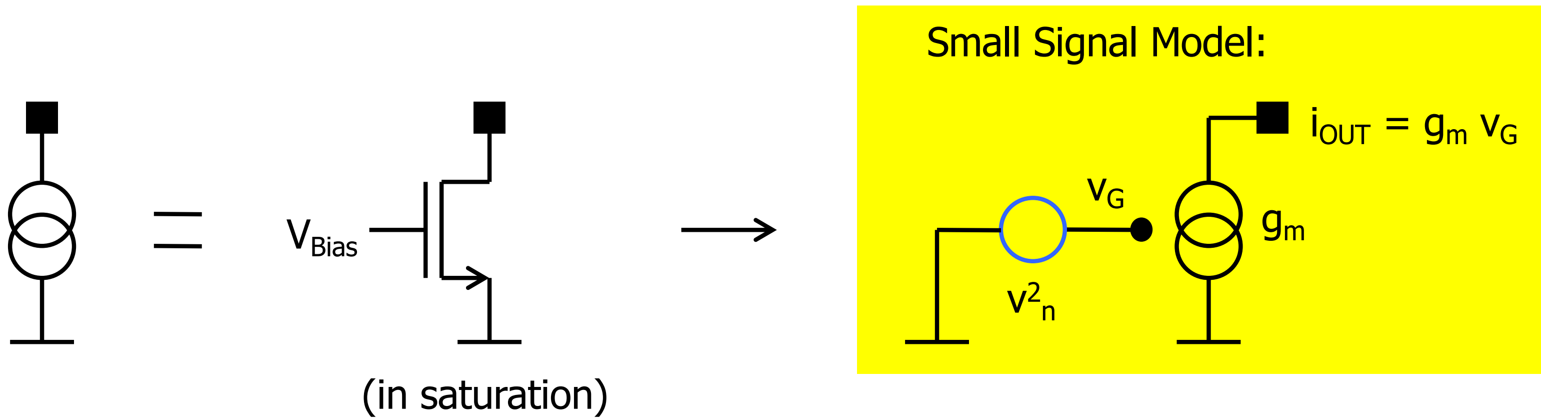
CURRENT SOURCES

MOS as Current Source



- Due to input voltage noise source, we get at the output

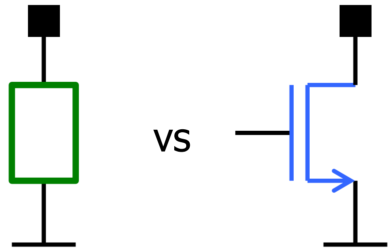
$$i^2_{\mathrm{OUT}} = g^2_m v^2_{\mathrm{G}} = g^2_m v^2_{\mathrm{n}} = g^2_m \frac{4kT\,\gamma}{g_m} = 4kT\,\gamma g_m$$

as before.

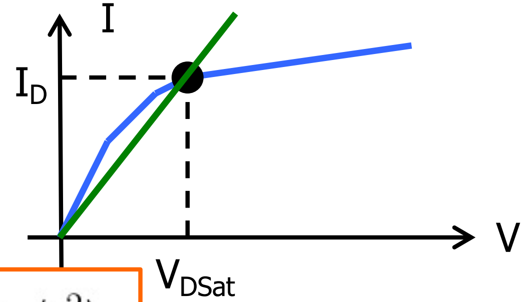
For a *low (current) noise current source* we need *small* g_m

(MOS vs. Resistor)

- For a current I_0, what gives lower noise: MOS or resistor ?
  - Assume we operate the MOS in s.i. just at edge of saturation:



- R: $\langle i^2 \rangle_R = \frac{4kT}{R} = 4kT \frac{I_D}{V_{DSat}}$
- MOS: $I_D = \frac{\beta}{2}(V_{GS} - V_{th})^2(1 - \lambda V_{DS})$



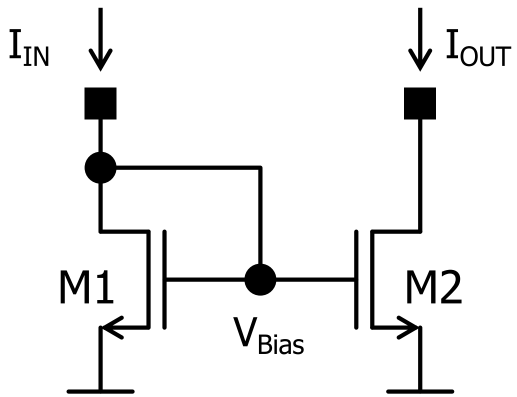
$$V_{DSat} = V_{GS} - V_{th} = \sqrt{\frac{2I_D}{\beta}}$$

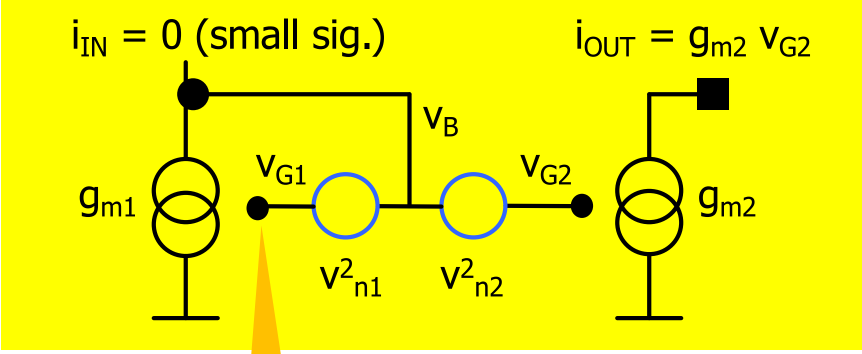
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \sqrt{2I_D\beta} = \frac{2I_D}{V_{DSat}}$$

$$\langle i^2 \rangle_{MOS} = 4kT\,\eta\, g_m = 4kT\,\eta \frac{2I_D}{V_{DSat}} = 2\eta \times \langle i^2 \rangle_R$$

- MOS is slightly *worse*, but has *much* lower output resistance
  - Also, current cannot be varied with fixed R
  - At higher voltage, R is larger and its noise decreases.
  - R has *no 1/f* noise!
  - Consider this 'old style' approach for very low noise…

Noise in the Simple Current Mirror





$0 = i_{IN} = v_{G1}\, g_{m1} \rightarrow v_{G1} = 0$

1. Left noise source: $v_{G1} = 0 \rightarrow v_B^2 = v_{n1}^2 = \dfrac{4kT\,\gamma}{g_{m1}}$

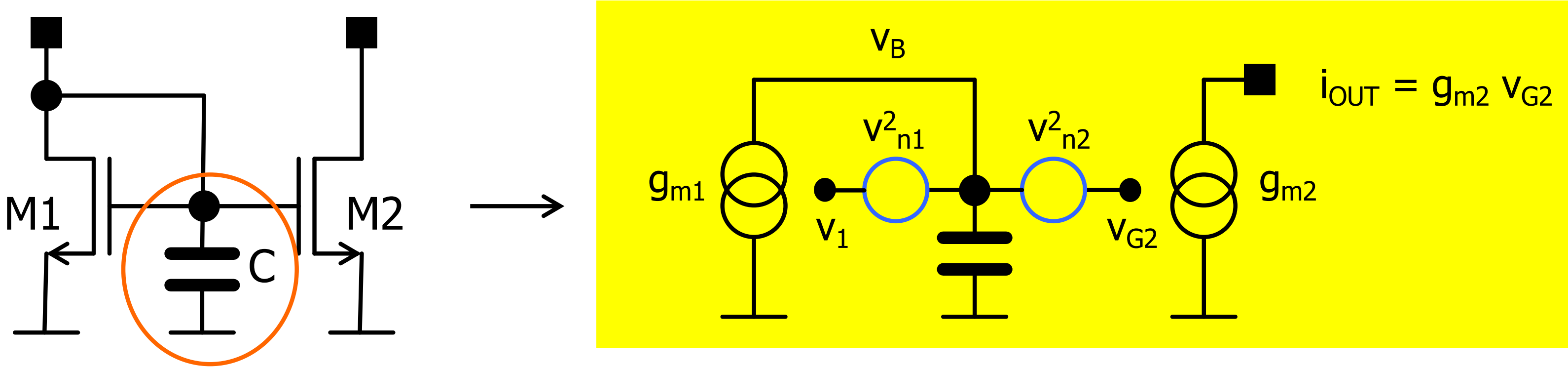
2. Right noise source: $v_{G2}^2 = v_B^2 + v_{n2}^2 = \dfrac{4kT\,\gamma}{g_{m1}} + \dfrac{4kT\,\gamma}{g_{m2}}$

Sum: $i_{OUT}^2 = v_{G2}^2\; g_{m2}^2 = g_{m2}\, 4kT\,\gamma \left(1 + \dfrac{g_{m2}}{g_{m1}}\right)$

current noise of M2

- For $g_{m1} = g_{m2}$ (1:1 Mirror), noise $\sqrt{i_{OUT}^2}$ increases by √2
- For $g_{m1} >> g_{m2}$ (N:1 Mirror), noise is small
- For $g_{m1} << g_{m2}$ (1:N Mirror), noise is large. **DO NOT USE**

Improving the Current Mirror with Decoupling



- The first component is multiplied by

$$\frac{1}{1+\frac{C^2\omega^2}{g_{m1}^2}}$$

so that we get overall

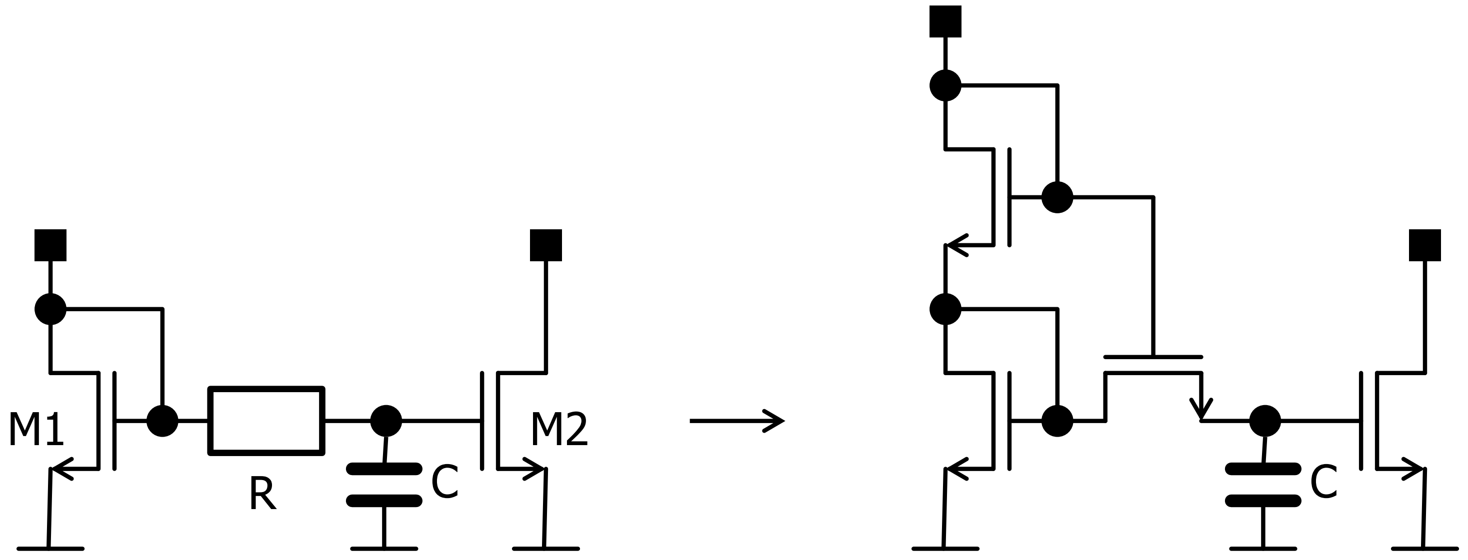
$$g_{m2}\,4kT\,\gamma\left(1+\frac{g_{m2}}{g_{m1}}\,\frac{1}{1+\frac{C^2\omega^2}{g_{m1}^2}}\right)$$

Noise of the input MOS is cut away above $\omega = g_{m1}/C$

```
In[41]:= EQ1 = gm1 v1 + vb s C == 0;
         EQ2 = v1 - vb == vn;

In[43]:= Eliminate[{EQ1, EQ2}, v1]

Out[43]= gm1 vn == (-gm1 - C s) vb

In[44]:= Solve[%, vb] // First

Out[44]= {vb -> -gm1 vn/(gm1 + C s)}

In[45]:= VB[s_] = vb /. %

Out[45]= -gm1 vn/(gm1 + C s)

In[46]:= VB[i ω] Conjugate[VB[i ω]]

Out[46]= gm1^2 vn^2/(gm1^2 + C^2 ω^2)
```

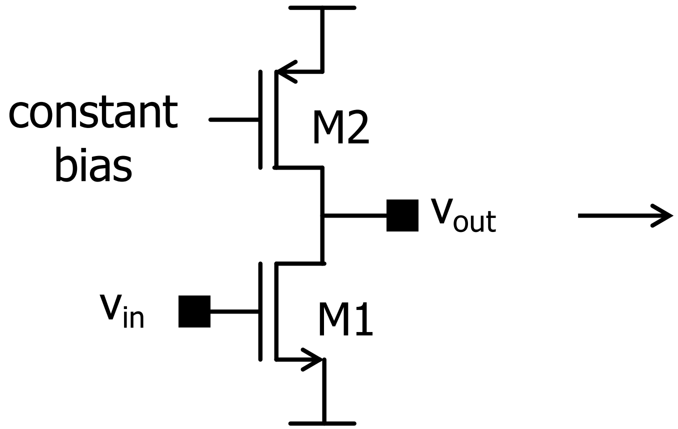
(One Possible further Improvement)

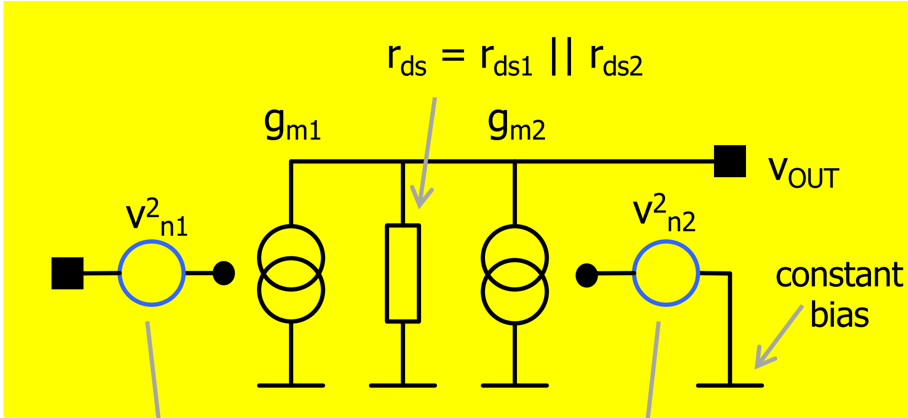
- An additional resistor R can be used to lower the bandwidth of the filter from g_{m1}/C to $1/RC$
- Note that R also adds its own noise...
- R can be implemented as a MOS with proper bias…



AMPLIFIER

Gain Stage





Treat noise sources independently!

- Output noise:

```
vout2 = vout2in + vout2load /.
  {vn1² → 4 k T γ / gm1, vn2² → 4 k T γ / gm2}
```

$4\,(gm1 + gm2)\,k\,rds^2\,T\,\gamma$

```
EQ1 = gm1 vn1 + vout/rds == 0
Solve[EQ1, vout] // First
{vout → -gm1 rds vn1}
vout2in = vout² /. %
gm1² rds² vn1²
```

```
EQ2 = vout/rds + gm2 vn2 == 0
Solve[EQ2, vout] // First
{vout → -gm2 rds vn2}
vout2load = vout² /. %
gm2² rds² vn2²
```

- Referred to input (divide by gain):

```
gain = gm1 rds; vin2 = vout2 / gain²
```

$$\frac{4kT\,\gamma}{g_{m1}}\left(1+\frac{g_{m2}}{g_{m1}}\right)$$

Gain of input MOS

Input MOS must have *high* g_{m1}
Bias Source must have *low* g_{m2}

The same using Current noise

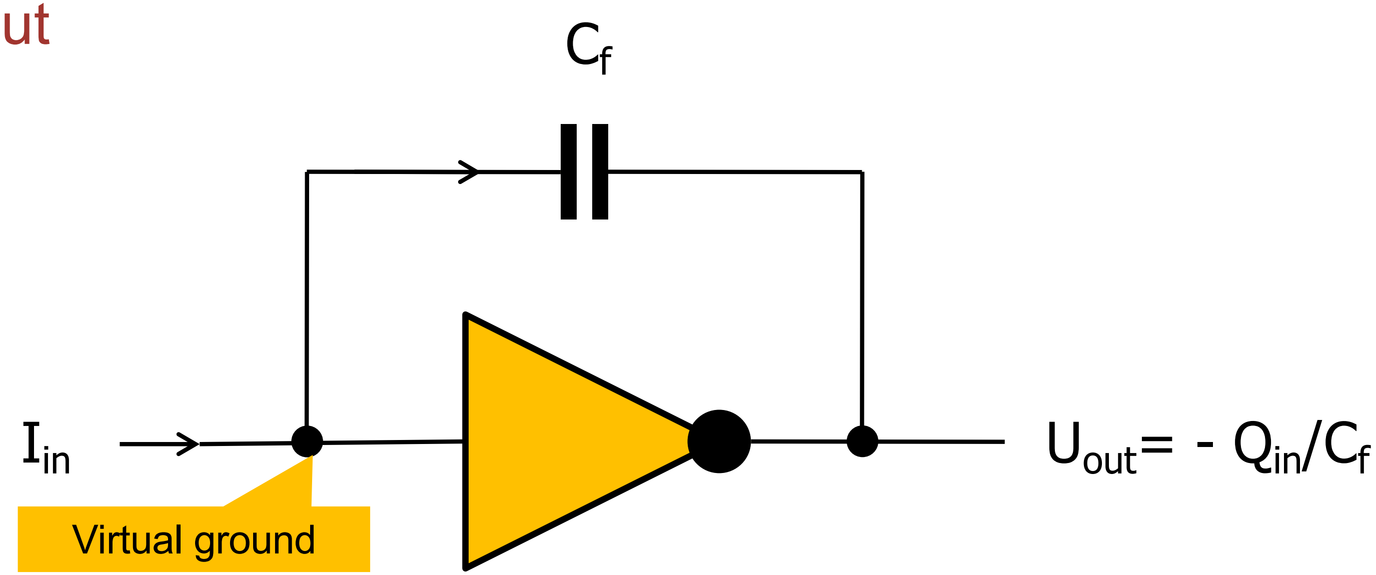
Use drain current sources instead of gate noise voltage

```
In[13]:= EQ = vin gm1 + vout/rds + in1 + in2 == 0; (* in1 and in2 are drain current noise of Min and Msource *)

In[14]:= Solve[EQ /. in1 → 0 /. in2 → 0, vout] // First
Out[14]= {vout → -gm1 rds vin}

In[15]:= voltagegain = vout/vin /. %
Out[15]= -gm1 rds

In[19]:= Solve[EQ /. vin → 0 /. in2 → 0, vout] // First
Out[19]= {vout → -in1 rds}

In[20]:= OutNoiseSquare1 = vout^2 /. %
Out[20]= in1^2 rds^2

In[21]:= Solve[EQ /. vin → 0 /. in1 → 0, vout] // First
Out[21]= {vout → -in2 rds}

In[22]:= OutNoiseSquare2 = vout^2 /. %
Out[22]= in2^2 rds^2

In[24]:= OutNoiseSquare = OutNoiseSquare1 + OutNoiseSquare2 // Simplify
Out[24]= (in1^2 + in2^2) rds^2

In[27]:= OutNoiseSquare /. {in1^2 -> 4 k T γ gm1, in2^2 -> 4 k T γ gm2} // Simplify
Out[27]= 4 (gm1 + gm2) k rds^2 T γ

In[28]:= InNoiseSquare = % / voltagegain^2
Out[28]= 4 (gm1 + gm2) k T γ / gm1^2
```

CHARGE AMPLIFIER

The Charge Amplifier

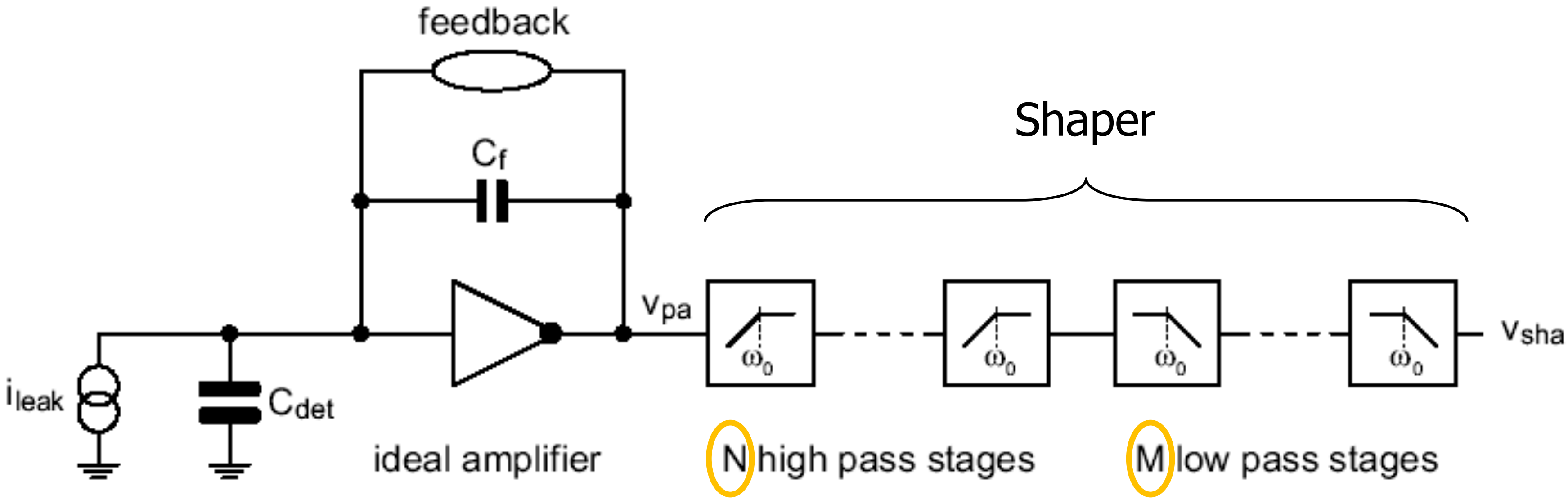
- The amplifier with feedback generates a *virtual ground* at its input



- Current (flowing charge) cannot stay on the input node (because the voltage is fixed) and must flow onto C_f
- The total input charge is the integral over I_{in}
- Therefore $Q_{in} = \int I_{in}\, dt = Q_f = U_f\, C_f \quad \rightarrow U_{out} = -U_f = -Q_{in}/C_f$

Classical System have a Filter = Shaper

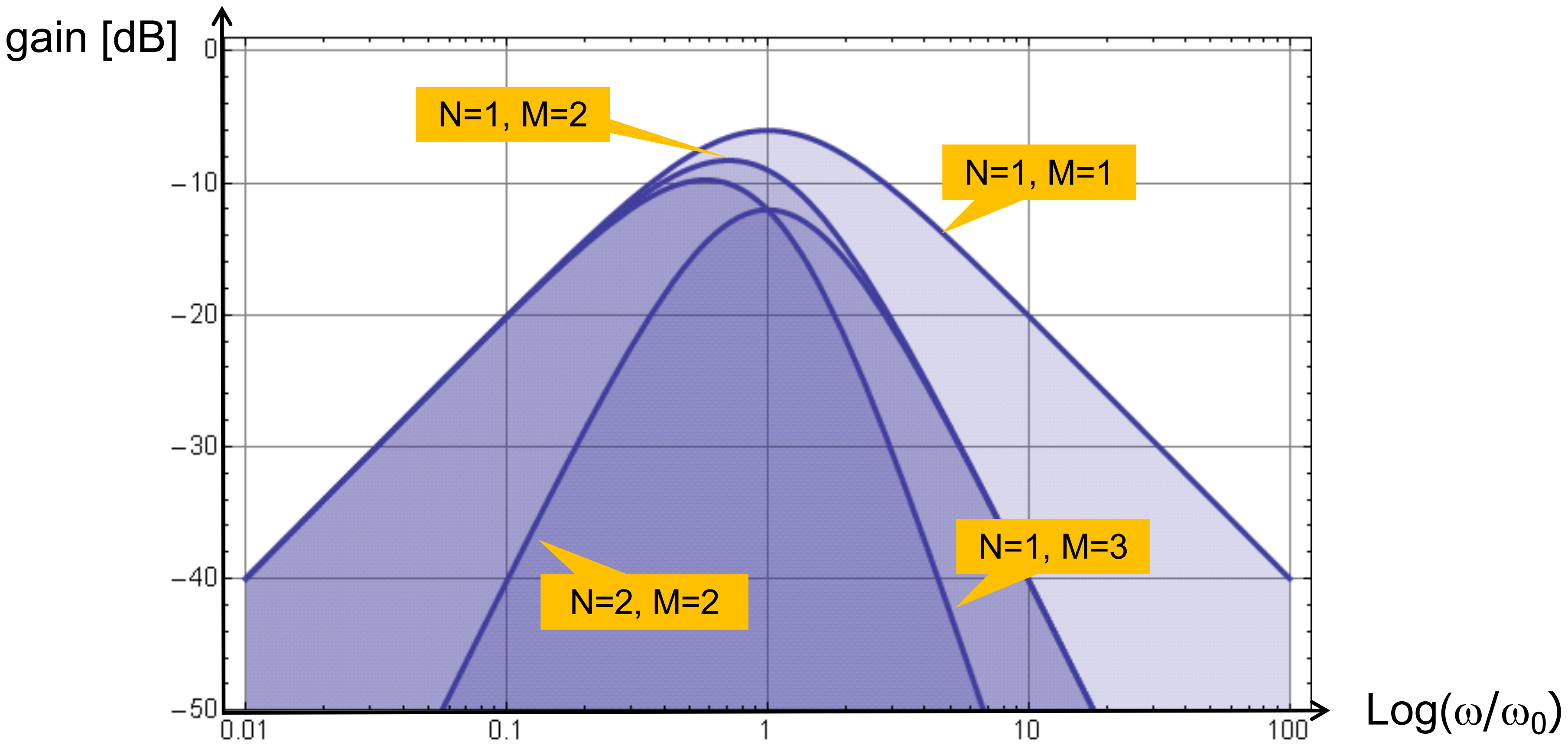
- Filter are added for pulse shaping & noise reduction:
  - *High pass stages* eliminate DC components & low freq. noise
  - *Low pass stages* limit bandwidth & therefore high freq. noise



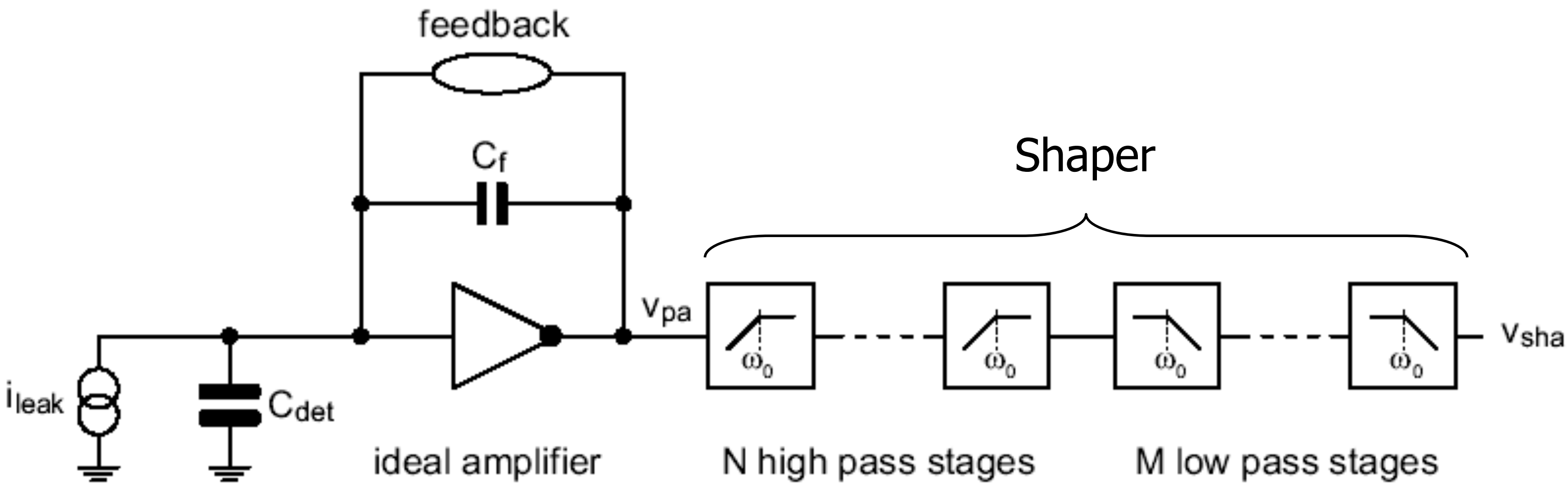
- Due to its output shape (see later), this topology is often called a 'Semi Gaussian Shaper'
- Nearly always N = 1. Often M = 1, sometimes M up to 8

Frequency Behaviour of Shaper

- Low and High frequencies are attenuated
- Corner frequency (here: 1) is transmitted best
- Bode Plot (log/log) of transfer characteristic:



What is the output signal?



- For a delta current pulse, the output voltage v_{pa} is a step function
- This has a Laplace-Transform ~1/s
- The transfer functions of the high / low pass stages multiply to:

$$\mathcal{L}^{(N,M)}(s) = \frac{1}{s}\left(\frac{s\tau}{1+s\tau}\right)^N \left(\frac{1}{1+s\tau}\right)^M = \frac{\tau^N s^{N-1}}{(1+s\tau)^{N+M}}$$

Pulse shape after shaper

- The time domain response is the inverse Laplace transform.
- The Laplace integral can be solved with residues:
  There is an (N+M)-fold pole at -1/τ

$$\begin{aligned} f^{(N,M)}(t) &= \operatorname{Res} \left. \frac{\tau^N s^{N-1} \mathrm{e}^{st}}{(1+s\tau)^{N+M}} \right|_{s=-1/\tau} \\ &= \frac{\tau^N}{(N+M-1)!} \lim_{s\to -\frac{1}{\tau}} \frac{\mathrm{d}^{N+M-1}}{\mathrm{d}s^{N+M-1}} \left[\frac{s^{N-1}\mathrm{e}^{st}}{(1+s\tau)^{N+M}} \left(s+\frac{1}{\tau}\right)^{N+M} \right] \\ &= \frac{1}{(N+M-1)!} \left(\frac{t}{\tau}\right)^M \sum_{i=0}^{\infty} \frac{(-\frac{t}{\tau})^i}{i!} \frac{(M+i+N-1)!}{(M+i)!} \qquad (1.42) \end{aligned}$$

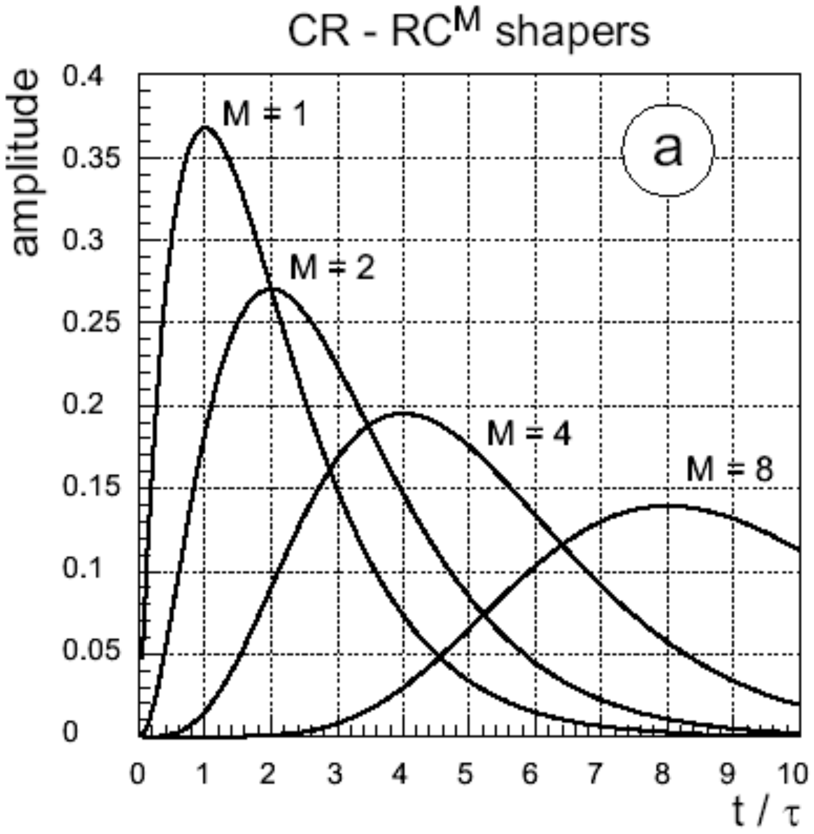
- For only ONE high pass section (N=1), this simplifies to:

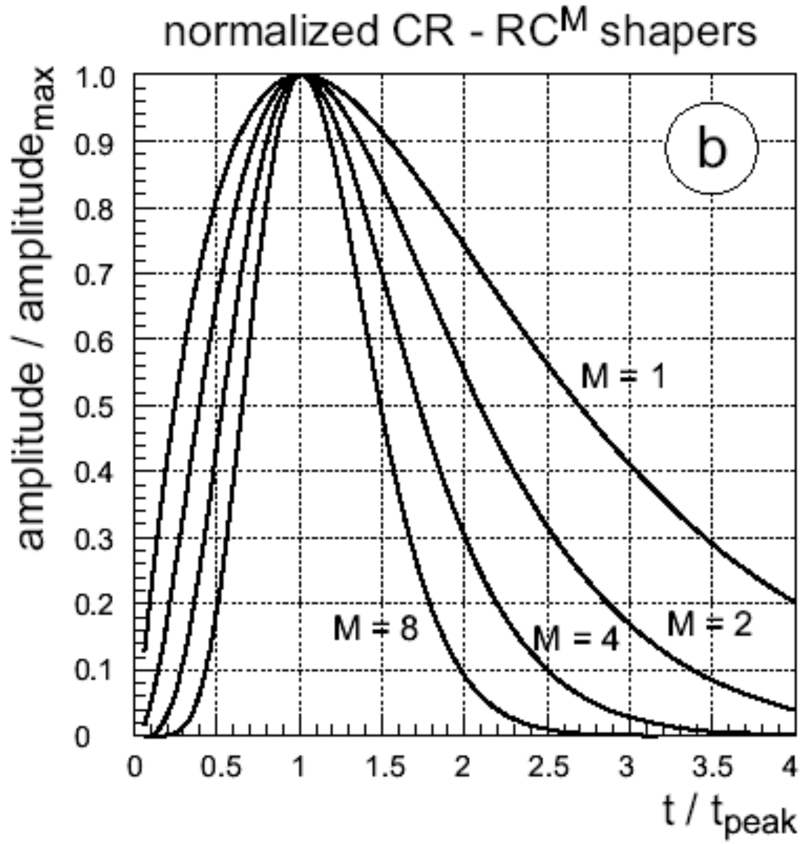
$$f^{(1,M)}(t) = \frac{1}{M!} \left(\frac{t}{\tau}\right)^M \mathrm{e}^{-t/\tau} \qquad t_{\mathrm{peak}}^{(1,M)} = M\tau = \frac{M}{\omega_0} \qquad f_{\max}^{(1,M)} = \frac{1}{M!}\left(\frac{M}{\mathrm{e}}\right)^M$$

$f_{max}^{(1,1)} = 1/e$

Pulse Shapes for N=1 (Only ONE High – Pass)

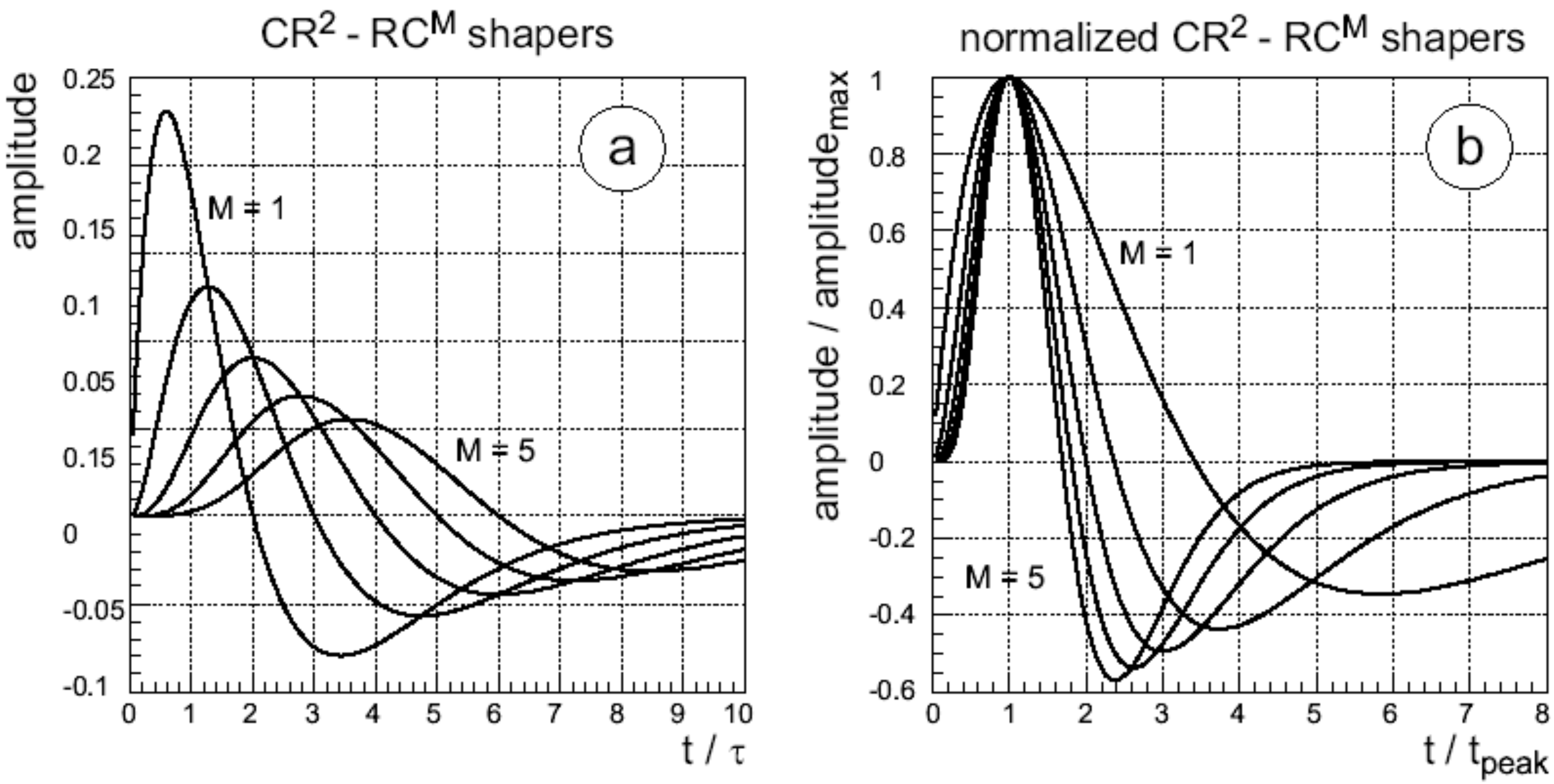
- Pulses from higher order are slower. To keep peeking time, τ of each stage must be decreased
- Right plots shows normalized pulses (same peak amp. & time)
- For high orders, pulses become narrow (width / peaking time), this is good for high pulse rates!





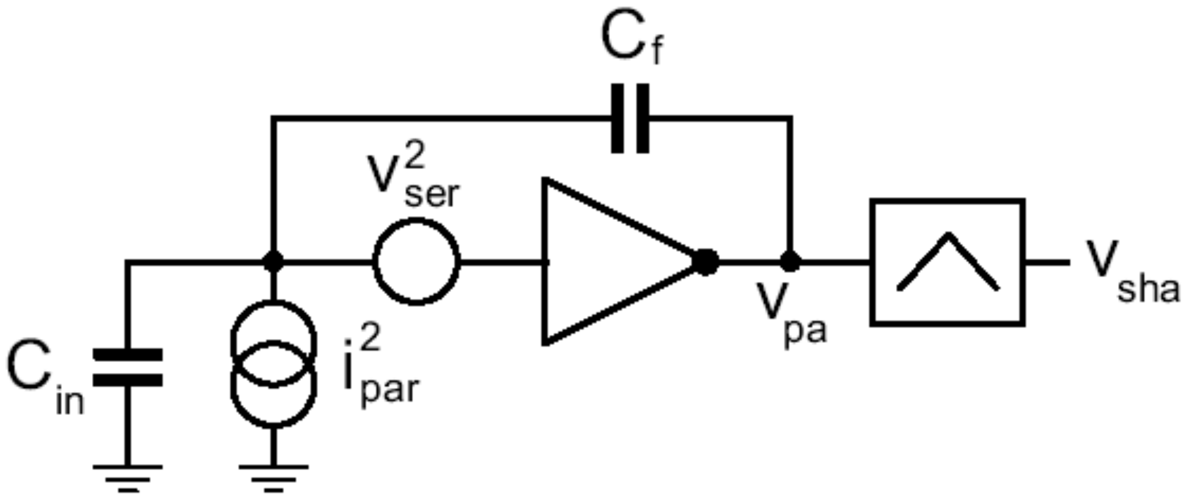
(Pulse Shapes for N=2)

- This gives an undershoot which is often undesirable → N=1.
  - But: The zero crossing time is *independent* of amplitude.
    It can be used to measure the pulse arrival time with no time walk



Noise calculation: Noise sources

- Equivalent circuit with (ideal) amplifier, input capacitance, feedback capacitance and (dominant) noise sources:



- Spectral densities of noise sources:

$$\text{serial noise voltage}: \quad \frac{\mathrm{d}\langle v^2(f)\rangle}{\mathrm{d}f} = V_0 + V_{-1} f^{-1}$$

$$\text{parallel noise current}: \quad \frac{\mathrm{d}\langle i^2(f)\rangle}{\mathrm{d}f} = I_0$$

white (channel)

1/f noise (MOS)

white (leakage)

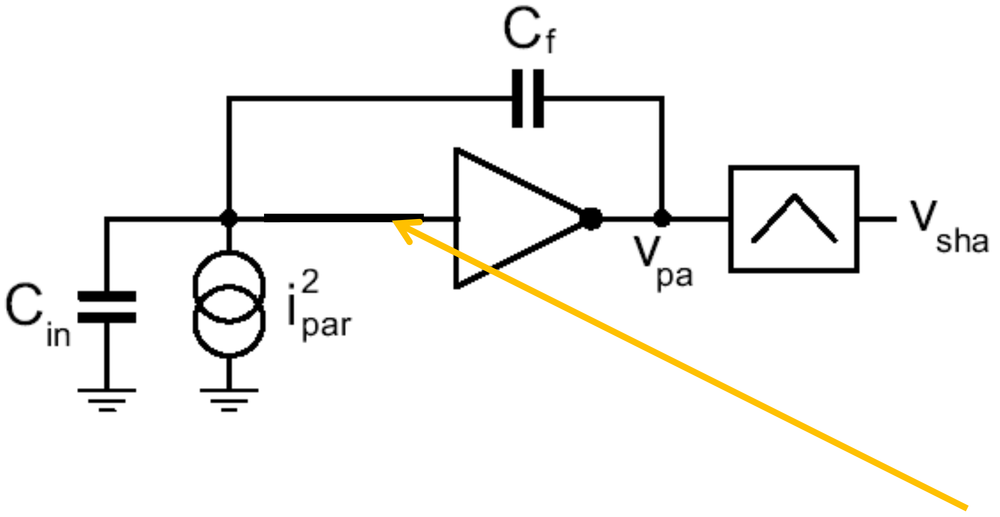
What is the total noise at the output ?

- Recipe (again):
  1. Calculate what effect a voltage / current noise *of a frequency f* at the input has at the output
  2. For each noise source: Integrate over all frequencies (with the respective densities)
  3. Sum contributions of all noise sources

- This yields the total rms voltage noise at the output

- Then compare this to a ‘typical’ signal.
  It is common to use *one* electron at the input as reference.

Parallel Noise Current



- We assume a perfect virtual ground at the amplifier input

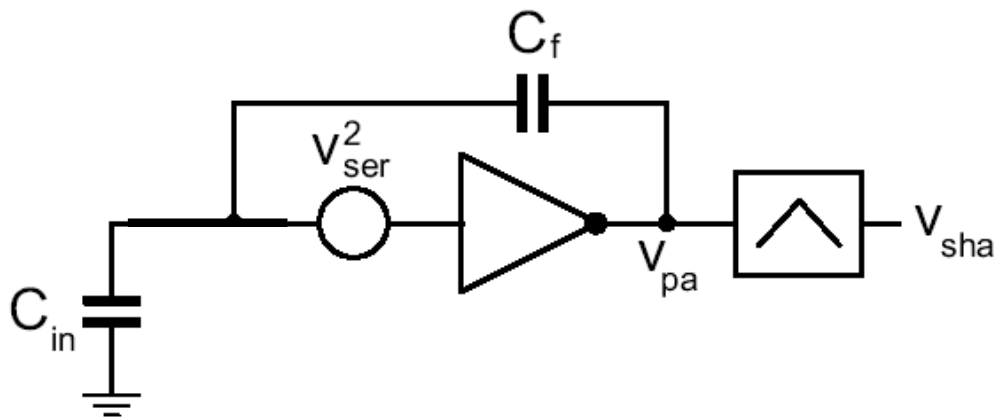
→ No charge can the go to C_{in} (voltages are fixed)

→ Noise current must flow through C_f: $v_{out} = i_{in} \times Z_{Cf}$

$$\text{parallel noise}: \quad \frac{d\langle v_{pa}^2(\omega)\rangle}{d\omega} = \frac{d\langle i_{par}^2(\omega)\rangle}{d\omega}\frac{1}{(\omega C_f)^2} = \frac{I_0}{2\pi}\frac{1}{(\omega C_f)^2}$$

(note the change of the frequency variable from ν to ω)

Serial Noise Voltage



- Output noise is determined by the capacitive divider made from C_f and C_{in}: $v_{ser} = v_{pa} \times Z_{Cin} / (Z_{Cin}+Z_{Cf})$

or:

$$v_{\mathrm{pa}}^2 = v_{\mathrm{ser}}^2 \left(\frac{C_{\mathrm{in}} + C_{\mathrm{f}}}{C_{\mathrm{f}}} \right)^2$$

Therefore:

$$\text{serial noise}: \quad \frac{\mathrm{d}\langle v_{\mathrm{pa}}^2(\omega) \rangle}{\mathrm{d}\omega} \approx \left(V_{-1}\omega^{-1} + \frac{V_0}{2\pi} \right) \left(\frac{C_{\mathrm{in}}}{C_{\mathrm{f}}} \right)^2$$

$(C_{in}=C_{det} + C_{preamp} + C_{parasitic})$

Total *Output* Noise (after the amplifier)

- In total, the output noise can be written as a sum of contributions with different frequency dependence:

$$\frac{\mathrm{d}\langle v_{\mathrm{pa}}^2(\omega)\rangle}{\mathrm{d}\omega} = \sum_{k=-2}^{0} c_k \omega^k$$

frequency dependence is here

with

$$c_{-2} = \frac{I_0}{2\pi C_{\mathrm{f}}^2}, \quad c_{-1} = V_{-1}\frac{C_{\mathrm{in}}^2}{C_{\mathrm{f}}^2} \quad \text{and} \quad c_0 = V_0\frac{C_{\mathrm{in}}^2}{2\pi C_{\mathrm{f}}^2}.$$

leakage (white)

MOS gate (1/f)

MOS channel (white)

from leakage current I_{leak}: $I_0 = 2qI_{\mathrm{leak}}$

from transistor channel noise: $V_0 = \frac{8}{3}\frac{kT}{g_m}$

from 1/f noise: $V_{-1} = \frac{K_{\mathrm{f}}}{C_{\mathrm{ox}}WL}$

Noise Transfer Function

- (N,M) - Shaper transfer function: $H^2_{N,M}(\omega) = A^2 \frac{\left(\frac{\omega}{\omega_0}\right)^{2N}}{\left(1 + \left(\frac{\omega}{\omega_0}\right)^2\right)^{N+M}}.$

- 'Filtered' noise at the output of the shaper:

$$\langle v^2_{\mathrm{sha}} \rangle = \int_0^\infty H^2_{N,M}(\omega)\,\mathrm{d}\langle v^2_{\mathrm{pa}}(\omega)\rangle = \sum_{k=-2}^{0} \int_0^\infty c_k\, \omega^k H^2_{N,M}(\omega) \mathrm{d}\omega$$

$$= \frac{A^2}{2} \frac{1}{\Gamma(N+M)} \sum_{k=-2}^{0} c_k\, \omega_0^{k+1} \Gamma\left(N + \frac{k+1}{2}\right) \Gamma\left(M - \frac{k+1}{2}\right)$$

$$\Gamma(z+1) = z\Gamma(z)$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi},$$

- For simplest shaper (N=M=1),
  Squared rms noise voltage at the shaper output:

$$\langle v^2_{\mathrm{sha}} \rangle = A^2 \frac{\pi}{4} \left(\frac{c_{-2}}{\omega_0} + \frac{2}{\pi} c_{-1} + \omega_0\, c_0 \right)$$

Calculation of ENC

- The **equivalent noise charge, ENC** is the (rms) noise at the output of the shaper expressed in Electrons input charge, i.e. divided by the 'charge gain'
- The 'charge gain' is (see before): Vmax = $q/C_f \times A \times 1/e$

charge of 1 electron (1.6e-19C)

Shaper dc gain

Peak amplitude for N=M=1

leakage gives noise for slow shaping

1/f – noise cannot be reduced by charging shaping time

$$\mathrm{ENC}^2_{\mathrm{CR-RC}} = \frac{\langle v^2_{\mathrm{sha}} \rangle}{V^2_{\mathrm{max}}} = \frac{\mathrm{e}^2}{4q^2}\left(\frac{\tau}{2} I_0 + \frac{1}{2\tau} V_0 C^2_{\mathrm{in}} + 2\,V_{-1} C^2_{\mathrm{in}}\right)$$

C_{in} is bad for **fast** shaping. Reducing V_0 requires large g_m

Noise contributions

- Real noise contributions for the coefficients I_0, V_0, V_{-1}:

from leakage current I_{leak}: $I_0 = 2qI_{\text{leak}}$

from transistor channel noise: $V_0 = \frac{8}{3}\frac{kT}{g_m}$

from 1/f noise: $V_{-1} = \frac{K_f}{C_{\text{ox}}WL}$

- For a 0.25µm technology (C_{ox}=6.4 fF/µm^2, K_f=33×10^{-25} J, L=0.5µm, W=20µm) and C_{in}=200fF, I_{leak}=1nA and τ=50ns, g_m=500µS (typical LHC pixel detector):

$$\left(\frac{\text{ENC}}{\text{e}^-}\right)^2 = 115 \cdot \frac{\tau}{10\,\text{ns}} \cdot \frac{I_{\text{leak}}}{1\,\text{nA}} \quad \rightarrow 575$$

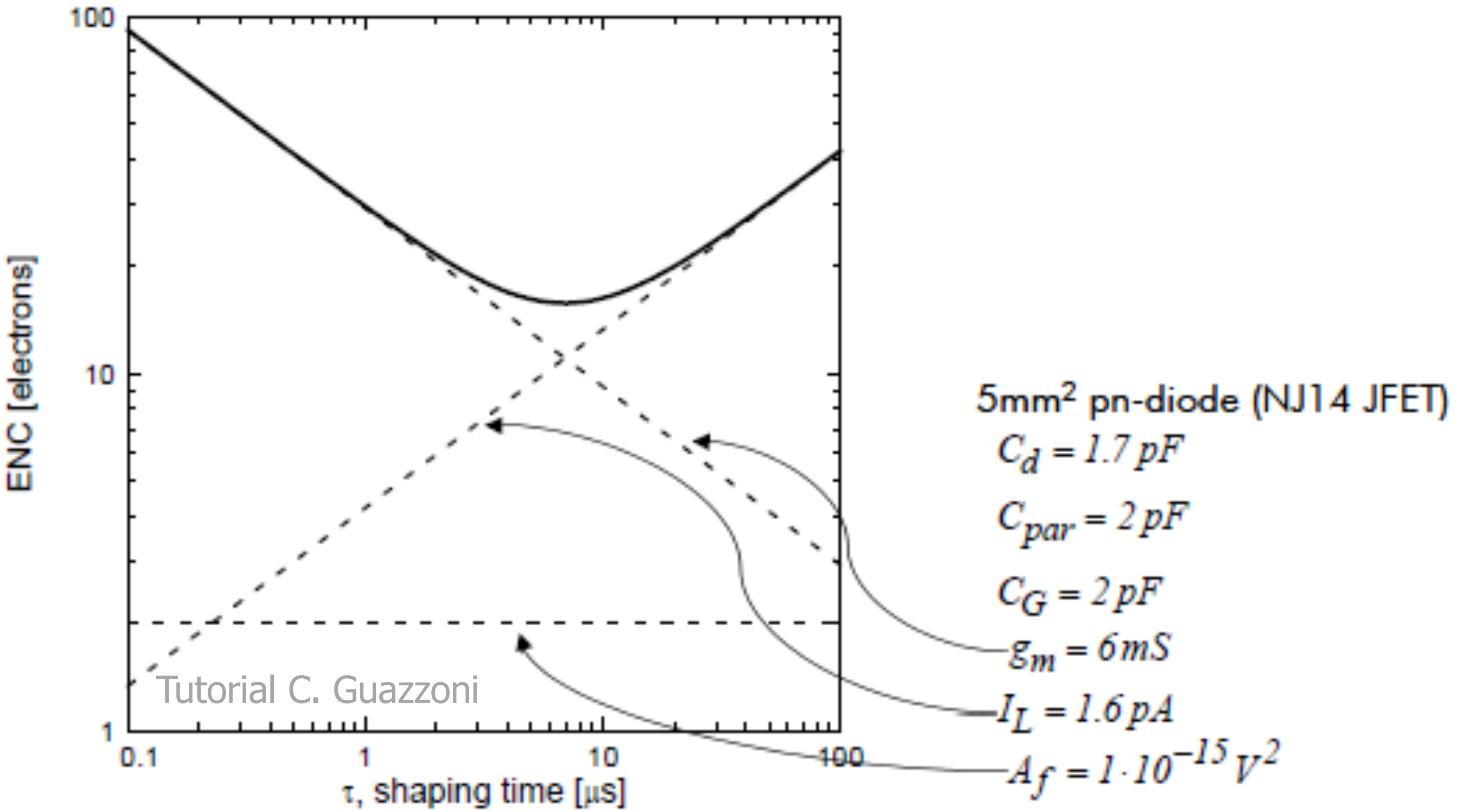
$$+ 388 \cdot \frac{10\,\text{ns}}{\tau} \cdot \frac{\text{mS}}{g_m} \cdot \left(\frac{C_{\text{in}}}{100\,\text{fF}}\right)^2 \quad \rightarrow 621$$

$$+ 74 \cdot \left(\frac{C_{\text{in}}}{100\,\text{fF}}\right)^2 . \quad \rightarrow 296$$

ENC=40 e$^-$

Noise vs. Shaping Time

- Long shaping: leakage noise contributes more
- Short shaping: Amplifier white noise, worsened by C_{Det}
- Always: Amplifier 1/f noise, worsened by C_{Det}



MISC

- For large MOS, the gate can have a significant resistance which can add thermal noise. Must use layout with multiple gate contacts.
- The noise coefficient γ can increase (quite) a lot for very short channel MOS. In general do NOT use shortest MOS if you need low noise.
- Noise models are often not very reliable. In particular 1/f noise can be run dependent. You may want to include noise test structures (i.e. large MOS arrays of the geometry you use)

- Noise can also be treated in transient simulation (enable 'transient noise' button). Good for nonlinear systems (comparator) and 'quick look'. Large simulation effort due to small time steps (high freq. noise components).
Provides less understanding where noise comes from…

Summary

- Resistor, MOS and Diodes (and BJTs) have noise
- Noise is described by its spectral density
- Noise contributions are propagated through the circuit with the respective frequency transfer function

- Total RMS noise is the integral over the spectra

- Current sources require MOS with low g_m.
  - This leads to larger saturation voltages…
- Amplifiers require MOS with large g_m

- Low bandwidth → Low Noise
- Filters limit bandwidth and thus reduce noise, but also decrease the signal…